OBTAINING ORTHOMETRIC HEIGHTS WITH HIGH ACCURACY BY GPS OBSERVATIONS OVER SMALL AREAS IN EGYPT

BY

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يعمل ال GPS بسرعة ودقة عاليتين بالنسبة الى الطرق التقليدية. يعطينا ال GPS مناسيب فوق النموذج الرياضى للأرض WGS84 (ارتفاعات جيوديسية) ولكن المناسيب المعمول بها مرجعها متوسط منسوب سطح البحر أو مايسمى بالجيويد (ارتفاعات ارثومترية) . لذا فان مسألة تحويل الارتفاعات من WGS84 الى ارتفاعات منسوبة الى الجيويد لها أهمية عالية وهى محل نظر الباحثين يستخدم ال GPS فى الأونة الأخيرة فى كثير من الأعمال المساحية و المدنية. من هذه الأعمال تحديد المناسيب على أهميتها الكبرى فى كل مكان. فى هذا البحث يتم تحويل الارتفاعات الجيوديسية الى ارتفاعات ارثومترية عن طريق نموذج للجيويد . الأرصاد المستخدمة فى هذا البحث عبارة عن أرصاد GPS للارتفاعات الجيوديسية وأرصاد ميزانيات للارتفاعات الارثومترية ونموذجين للجيويد أحدهما محلى مصرى والاخر نموذج عالمى. أخذت الارصاد للبحث فى خمس مناطق مختلفة فى مصر وهى حلوان والعبور والضبعة ودمياط وتوشكا. وضعت نتائج البحث فى عشرة جداول ومثلت فى سبعة أشكال. أظهرت النتائج أنه يمكن الأعتماد على ال GPS لايجاد المناسيب فى المناطق الصغيرة بالاستعانة بنموذج للجيويد.

Abstract

The Global Positioning System (GPS) provides the surveyor with three-dimensional coordinates with respect to the World Geodetic System 1984 (WGS84). The heights obtained from GPS are above an ellipsoidal model of the earth, WGS84. However, it is unlikely that ellipsoidal heights will ever be used for practical surveying, engineering or geophysical applications, as they have no physical meaning. Therefore, it will always necessary to transform GPS-derived ellipsoidal heights to orthometric heights. Conversion of GPS heights requires a high-resolution geoid height model. In this context, orthometric heights can be obtained without leveling by combining the geoidal undulations with the ellipsoidal heights derived from GPS. The goal of this research is to compare orthometric heights derived from GPS observations with orthometric heights obtained from spirit leveling observations over small areas in Egypt. For this purpose, five test areas were chosen, the first one is in Helwan city [12 stations], the second one is in Al-Obour city [7 stations], the third one is in Al-Dabaa city [16 stations], the fourth one is in Domiate port [8 stations] and the fifth one is in Toshkii [10 stations]. GPS

measurements, spirit-leveling measurements, were performed at all stations in each test area. The geoid undulations were computed using three techniques. One of them GPS/Leveling consists of computing the differences between the ellipsoidal and orthometric heights, the second technique was by calculating the undulations from the gravimetric Geoid-95 and the third technique was by deriving the geoidal undulations from the global geopotential earth model, EGM96. The analysis of the obtained results shows that with available accurate geoid, the determination of geoidal heights is currently at least as accurate as the determination of ellipsoidal heights by GPS-phase measurements. This opens interesting possibilities for orthometric height determination in the mountains and in remote areas without traditional vertical control procedures.

1. Orthometric Heights from GPS and Precise Geoid

The geoid, loosely defined as the equipotential surface at mean sea level, is of considerable importance for the definition of a consistent height system. It is the reference surface for the system of geopotential numbers, from which most of the existing height systems can be derived [Heiskanen and Moritz, 1981]. Particularly, the commonly used system of orthometric heights has a simple definition in terms of the geoid. It is the distance from the geoid to the surface point measured along the plumb line (Figure-1). Despite of its great conceptual importance, the geoid and its determination has always been considered to be the domain of relatively few specialists. This is due to the fact that the geoid is not explicitly needed in leveling operations. Leveled height differences can be easily transformed into orthometric height differences by adding a certain correction term, due to the influence of local gravity field [Nassar, 1977]. This situation has changed with the advent of the NAVSTAR Global Positioning System (GPS). When using GPS in the interferometric or phase measurement mode, this system gives three-dimensional coordinates or coordinate differences with high accuracy. The Cartesian coordinates (X, Y, Z) can be transformed into ellipsoidal coordinates (ϕ, λ, h). The relation between ellipsoidal height (h), orthometric height (H), and geoidal height (N) is shown in Figure (1).

The orthometric heights of the stations can be then determined using the formula:

$$H = h - N \tag{1}$$

Where:

H is the orthometric height,

h is the ellipsoidal height determined by GPS techniques, and

N is the geoidal height above the ellipsoid.

Instead, the change in orthometric height over a GPS baseline (A to B) is determined by using the corresponding change in geoid-ellipsoid separation [Featherstone, et. al, 1998], as:

$$H_A - H_B = h_A - h_B - (N_A - N_B) \tag{2}$$

$$\Delta H_{AB} = \Delta h_{AB} - \Delta N_{AB} \tag{3}$$

Since h can be obtained with high accuracy from GPS-interferometry, the question arises whether N can be determined with comparable accuracy. This would provide the capability to determine orthometric height without leveling [Steed and Holzangle, 1994]. Thus, coordinates obtained from NAVSTAR/GPS could be directly related to coordinates from the usual terrestrial methods. In addition, orthometric height in mountainous terrain and between points that are at large distances apart could be obtained more economically by using Equation (1) or (2). The authors believe that this will be the case because orthometric heights are physically meaningful, while ellipsoidal heights are a mathematical abstraction. Since bodies of standing water are in first approximation level surfaces, large irrigation and dam projects will always require the use of orthometric heights [Nassar, 1977]. The accuracy of the orthometric heights computed by Eq. (1) is dependent on the accuracy of the two components h and N. While the first one is derived from GPS measurements, the second one is obtained by computation using available astrogeodetic, gravimetric or any other data. The current accuracy of GPS-interferometry is about 1ppm for position, and 2-3 ppm for height for inter-station vectors between 10 and 70 km [Schwarz and Sideris, 1993]. It is expected that these accuracies can be improved by one order of magnitude through a better modeling of the orbit and the atmosphere [Rummel, 1992].

2. Geoidal Heights Using Gravimetric Technique

In its strict sense, the term gravimetric technique refers to solutions of the geodetic boundary value problem where the geoidal heights, N, are determined from gravity anomalies, Δg, on the boundary surface. The geoid determination implies the computations of its surface relative to a certain ellipsoid. This is expressed as the surface trend (geoid undulations N), and its slopes in two principal directions (meridian and prime vertical deflection components). The basic idea here is to split the geoid undulation N, into three components or parts, one per each type of gravity field data source.

Formally, the solution of the problem, in this case, can be written as [El Sagheer, 1995]:

$$N = N1 + N2 + N3 \tag{4}$$

Where:

N1 represents the contribution of the adopted geopotential model GM, as a general global trend.

N2 represents the contribution of the smoothed gravity anomalies Δg, as a regional trend.

N3 represents the contribution of the topographic local irregularities of the terrain, as a local residual trend.

The above three different contributions, for the geoid undulation.as an example, may be visualized by considering a typical geoidal profile of about 200km length in mountainous area, as diagrammatically illustrated in Figure (2). In this Figure, N1 represents the global trend or the long wavelength features of the geoid, which changes very smoothly. N2 represents regional and local geoid features with medium wavelength typically between 20 and 200 km. N3 represents the short wavelength features below 20 km that are caused by the topography, and which, in mountainous terrain, changes rapidly but usually have small amplitudes. Typical orders of magnitude in Equation (4) are meters for N1, decimeters for N2 and centimeters for N3 [Schwarz, et al. 1987].

The mathematical formulations, as well as the practical computations, of each individual component or wavelength, in Equation (4) are given in El Sagheer, 1995. A gravimetric geoid has been computed for the Egyptian region. The obtained results are presented in the form of a homogeneous grid of 5' interval. From the grid, the corresponding contour map, with reasonable scale and hence contour interval can be produced.

The accuracy of N as given by Equation (4) depends on the accuracy of its three components. Errors in N1 are mainly due to errors in the spherical harmonic coefficients. They are of long wavelength nature and can therefore not be eliminated by data given in the immediate area. Errors in N2 are strongly dependent on the accuracy and the average spacing of the gravity anomalies, i.e., on the resolution of the local gravity field that can be obtained from the given data. Errors in N3 are due to data spacing and reduction methods. While the data spacing is usually sufficient to model the high frequencies, inadequate reduction methods may introduce errors with longer periods. It has been assumed in the following that errors from, incorrect value of the mass of the earth, use of the spherical approximation, neglected atmospheric correction, and from datum errors are either completely or due to a large extent eliminated by differencing.

Again, although the accuracy of the derived undulation (Eq. 4) may be in the order of few meters, the relative accuracy of the undulation difference over a certain baseline (up to 100 km) can reach a value of 3 ppm, as mentioned before. This gives few decimeters accuracy, for instance yields an accuracy of 10 to 15 centimeter over a distance of about 50km. It has been found that a grid size of about 10km (5') for both gravity anomalies and topographic terrain models will satisfy the above analyzed accuracy.

3. Geoidal Heights Using Geopotential Earth Models Technique

If a gravimetric geoid model is unavailable or in low accuracy, as a situation in Egypt, in the region to be surveyed, the global geopotential earth model e.g. EGM96 can be used instead. For the geopotential models, the geoid heights are computed using the following spherical harmonic expansion [Rapp, 1982]:

$$N(r, \phi, \lambda)=(GM/r\gamma)\sum_{n=2}^{360} (a/r)^{n} \sum_{m=0}^{n} (\bar{C}_{nm} \cdot \cos m\lambda + \bar{S}_{nm} \cdot \sin m\lambda) \cdot \bar{P}_{nm} (\sin \phi) \qquad (5)$$

where γ is normal gravity at computation point, r is the geocentric radial distance of the computation point projected on the ellipsoid, a is the semi-major axis, ϕ is the latitude and λ is the longitude. Finally, Cnm and Snm are the harmonic coefficients and Pnm are the associated Legendre polynomials. A computer file of the geopotential coefficients is freely available, and geoid-WGS84-separation can be computed on a desktop computer using the routines of Rapp, 1982.

4. Description of the Local Test Areas

- The first local GPS network under consideration, located in Helwan, includes 12 GPS points. These points were established during 1996. Two Leica GPS dual frequency receivers were used in this network. Each receiver required for its operation one operator and one assistant. All points had both orthometric and ellipsoid height information available. A description of the used data points and the associated statistics, which covered an area of 3km x 3km, is given in Tables (1&2).
- The second local GPS network under consideration, is located at Al -Obour City, it includes a subset of 7 GPS points. These points were established during 1996. The same two Leica GPS receivers were used in this network. Also all points had both orthometric and ellipsoidal height information. The used data covered an area of 3.5 km x 8.5 km, for the second test area which is illustrated in Table (3) and the statistics in Table (4).
- The third local GPS network under consideration, is located at Al-Dabaa City, it includes a subset of 16 GPS points. These points were established during 1999. Three Leica GPS dual frequency receivers were used in this network. Also all points had both orthometric and ellipsoidal height information. The used data covered an area of 6 km x 8 km, for the third test area which is illustrated in Table (5) and the statistics in Table (6).
- The fourth local GPS network under consideration, is located at Domiate port, it includes a subset of 8 GPS points. These points were established during 2000. The same three Leica GPS receivers were used in this network. Also all points had both orthometric and ellipsoidal height information. The used data covered an area of 2.5 km x 4.5 km, for the fourth test area which is illustrated in Table (7) and the statistics in Table (8).
- The fifth local GPS network under consideration, is located at Toshkii project area, it includes a subset of 10 GPS points. These points were established during 2001. Three Trimble 4000SSI receivers were used in this network. Also all points

had both orthometric and ellipsoidal height information. The used data covered an area of 5.5 km x 4.5 km, for the fifth test area which is illustrated in Table (9) and the statistics in Table (10).

It is worthwhile to mention that, these networks were designed and observed by the authors. The used software was SKI and Geomatics. The observations were collected in the static-relative mode. One receiver occupied the reference or base station. The other receivers were occupying the other stations. The sessions were planned before taking the observations through the preparation module in the used software. The session at every station ranges between 20-40 minutes with 15 seconds sequential observation interval. The collected data were processed by the mentioned softwares to obtain the coordinates of the network stations relative to WGS84.

The tolerance in the reference station/s for the GPS work should not exceed some limits, otherwise the resulted base lines will be affected. The reference station for the first test area was O1, one of the first order triangulation stations of Egypt. O1 is already defined in WGS84 within the required precision before our work. Similarly, the reference station for the third test area was D8, one of the first order triangulation stations of Egypt. Station O5, one of the first order triangulation stations, is used as reference station for GPS work of the fifth test area. For the second and fourth test areas GPS observations were taken at the reference stations for 12 hours on 2 sessions. These 12 hours observations were processed as single point (Point Positioning) to give a reasonable, within the required precision, WGS84 coordinates. The manuals of the GPS receivers (e.g. Leica) mentioned that 6 hours are enough for this purpose. In all cases, the reference stations were accurate enough to produce accurate base lines (coordinate differences) whom the research will depend on. In all test areas, the orthometric heights are obtained from the nearest Bench Marks of the Egyptian Surveying Authority using the spirit levelling.

5.Obtained Results and Analysis of Orthometric Height Determination

Three tests corresponding to three different situations are described in this section. Firstly, let us analyze the three height data sets gathered in every test area: The ellipsoidal heights (h) were carefully obtained by GPS technique as described above. The orthometric heights (H) were all obtained by differential spirit leveling as previously described. The geoidal heights (N), also called geoid undulations, were obtained by interpolation within a grid of geoidal height values covering all of Egypt and computed by El Sagheer, 1995 [Geoid-95] and also from the global geopotential earth model EGM96.

First test area (Helwan): Table (1) shows the data and results of the first test area. Figure (3), for the first test area, illustrates the profile of the Geoid-95 geoidal heights (N_{Grav}) together with the GPS/leveling derived geoid profile (N_{GPS}) and the geoidal height derived from EGM96 (N_{EGM96}). The shape of the three profiles is basically the same, but with some obvious differences. These differences, indicates a shift between the three geoidal profiles. The statistics related to Figure (3) are given in Table (2). The difference between Geoid-95 and the GPS derived geoid (Δ1) has a mean value of 0.891 meter, with RMS equal to ±1.5 cm. Also, the difference between EGM96 and the GPS derived geoid (Δ2) have a mean value of 10.028 meter, with RMS equal to ±1.5 cm.

Second test area (Alobor): The same analysis has been done for the second test area, using data in Table (3), and presented in Figure (4). The shape of the three profiles is basically the same as in the first test area. The statistics related to Figure (4) are given in Table (4). The difference between Geoid-95 and the GPS derived geoid (Δ1) has a mean value of 0.899 meter, with RMS equal to ±1.4 cm. Also, the difference between EGM96 and the GPS derived geoid (Δ2) have a mean value of 9.300 meter, with RMS equal to ±1.5cm.

Third test area (Aldabaa): The same analysis has been done for the third test area, using data in Table (5), and presented in Figure (5). The shape of the three profiles is basically the same as in the first test area. The statistics related to Figure (5) are given in Table (6). The difference between Geoid-95 and the GPS derived geoid (Δ1) has a mean value of −15.253 meter, with RMS equal to ±0.3 cm. Also, the difference between EGM96 and the GPS derived geoid (Δ2) have a mean value of −1.261 meter, with RMS equal to ±0.4 cm.

Forth test area (Domiate): The same analysis has been done for the fourth test area, using data in Table (7), and presented in Figure (6). The shape of the three profiles is basically the same as in the first test area. The statistics related to Figure (6) are given in Table (8). The difference between Geoid-95 and the GPS derived geoid ($\Delta 1$) has a mean value of -14.712 meter, with RMS equal to ±1.8cm. Also, the difference between EGM96 and the GPS derived geoid ($\Delta 2$) have a mean value of -3.188 meter. with RMS equal to ±2.9cm.

Fifth test area (Toshkii): The same analysis has been done for the fifth test area, using data in Table (9), and presented in Figure (7). The shape of the three profiles is basically the same as in the first test area. The statistics related to Figure (7) are given in Table (10). The difference between Geoid-95 and the GPS derived geoid ($\Delta 1$) has a mean value of -9.831 meter, with RMS equal to ±0.5 cm. Also, the difference between EGM96 and the GPS derived geoid ($\Delta 2$) have a mean value of 0.513 meter, with RMS equal to ±0.5 cm.

In all cases, difference of undulations from the three data sources are very near to each others. curves are all parallel. So the equation $\Delta H = \Delta h - \Delta N$ can be used with good results and this is the subject of this research. On the contrary, The absolute values of the undulations from the different data sources are not equal. In two cases, (N_{Grav}) is nearer to (N_{GPS}) and (N_{EGM96}) is distant. In three cases, (N_{EGM96}) is nearer to (N_{GPS}) and (N_{Grav}) is distant. The reason may be that the gravimetric geoid does not represent the north of Egypt (Domiate and Aldabaa) and also the south of Egypt (Toshkii) because of lake of gravity data there. Reference stations of GPS work could also not be consistent.

The five-mentioned test areas are small, in the sense that the gravity effect on the observed height differences, in every case, can be neglected. The height differences are observed very carefully using spirit level and staffs. The closing errors in the five cases vary from 6 and 15 mm. These closing errors are corrected proportionally to the distances.

Future tests involving more geoid models may confirm or enhance this value. These tests will be performed in a near future. In general the gravimetric geoid [Geoid-95] and geopotential earth model geoid EGM96 look significantly parallel to the GPS derived geoid in all areas (see Tables 1, 3, 5, 7, 9 and Figures 3, 4, 5, 6, 7). Tests with other independent geoid solutions should demonstrate if these errors are in the GPS data or not.

To overcome this problem we must fix at least one station in the area under consideration with known values for orthometric, and ellipsoidal heights and compute the geoidal undulation at this station. By comparing this value with the interpolated gravimetric or EGM96 geoidal undulation at the same station, we can get the residuals (V/Δ1) or (V/Δ2). Subtracting these residuals (V/Δ1) or (V/Δ2) value from the other stations, we can improve the obtained results as illustrated in Tables (1), (3), (5), (7) and (9). From the analysis of these obtained results as illustrated in Tables (2), (4), (6), (8) and (10), we have good accuracy's for the computed orthometric heights which vary from – 2.7 cm to 2.3 cm. These results open interesting possibilities for orthometric height determination, in the mountains and in remote areas without traditional vertical control procedures. This can satisfy many practical applications, in addition to saving time, efforts and costs, especially in long distances where the spirit leveling will suffer from the accumulated errors.

6. Conclusion

As a result from the five test areas one can conclude that:

- Orthometric heights can be obtained by GPS and the geoid model with uncertainties ranging according to the used geoidal model, local or global one.
- More work should be done to improve the geoid models throughout the country and in testing GPS survey procedures.
- The presence of at least one known station of orthometric, ellipsoidal and gravimetrically or global geoidal undulation, improve the results and the range of accuracy varies from –2.7 cm to 2.3 cm.

- In terms of accuracy, leveling still yields better results than the proposed method, however, as computed geoid undulations and GPS positioning accuracy's improve. It is expected that GPS will become very competitive with leveling at lower order of accuracy`s.
- GPS differential height determination is an accepted accurate method, beside leveling, in regional crustal movement monitoring, since there is no need to compute orthometric heights for the GPS accurately observed ellipsoidal heights.
- Appropriate planning of the GPS survey and careful use of control points of high quality is required. A wide variety of mapping and engineering projects can be fulfilled with this kind of accuracy.
- Applying a quality assurance procedure to the GPS-derived orthometric heights by observing GPS at as many benchmarks as possible in and around the survey area. These data allow subsequent checks to be made on the typical accuracy of GPS-derived orthometric heights for each particular survey in each particular area.
- If the used geoid model is one homogeneous model and the orthometric height system (Bench Marks of ESA) is also one consistent system, so GPS work should have a unified datum. Unified GPS datum here means consistent reference stations all over the country. One can use reference station obtained from Point Positioning of 4 hours data, another one can use 8 hours data. The third one can use reference station which is tied to an IGS station/s ….etc. Many different values could be obtained for the same station, they can differ by meters. This should be regarded while using GPS in tying different distant areas to each others.
- In the case that every area will be treated separately, so the values under consideration are the differences as $\Delta H = \Delta h - \Delta N$ and just the GPS vectors or base lines should be accurate. One station with known orthometric height should be there.
- In small areas, where the two GPS receivers will not apart more than 10 Km from each others, Kinematic GPS technique could be used with almost the same accuracy instead of the Static mode. In such a case, Static mode duarte 20-40

minutes while Kinematic mode will duarte few minutes at every station. This will very much reduces the observation time.

In conclusion, leveling is slow, expensive and labor intensive; while GPS proposed method is fast, cost effective and simple.

7. References

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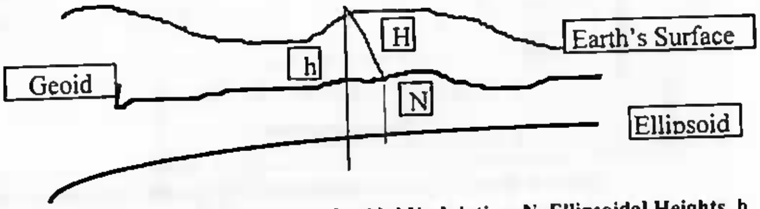

Figure 1: The Relationship between the Geoidal Undulation, N, Ellipsoidal Heights, h, and Orthometric Height, H.

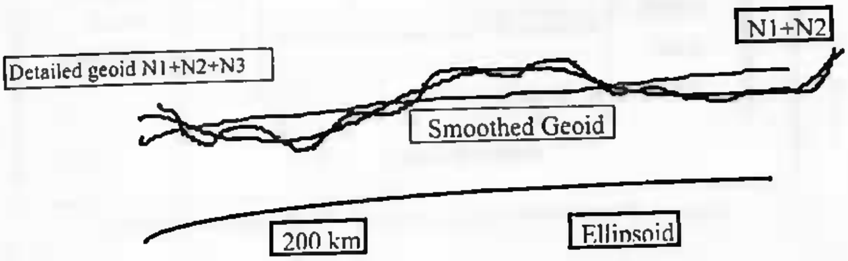

Figure 2: A Schematic Profile Showing the Contribution of Different Data, Representing the Earth's Gravity Field, to Local Geoid Determination.

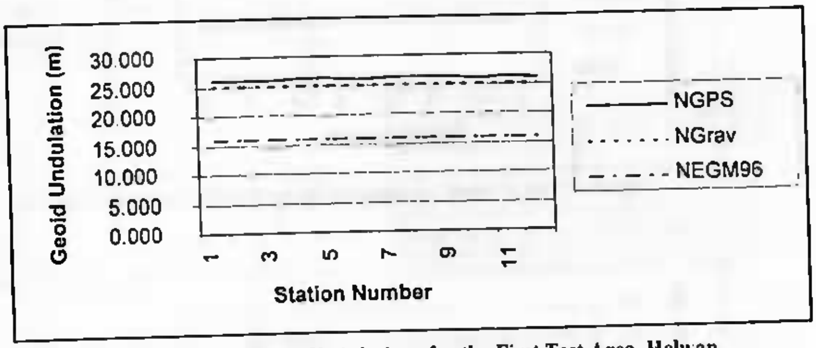

Figure 3: Plot of Geoid Undulations for the First Test Area, Helwan.

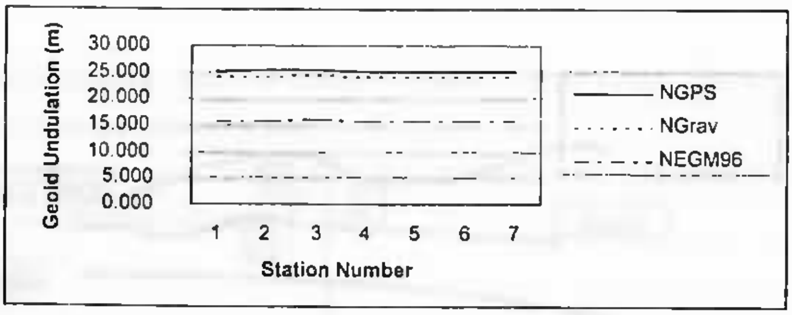

Figure 4: Plot of Geoid Undulations for the Second Test Area, Al Obour.

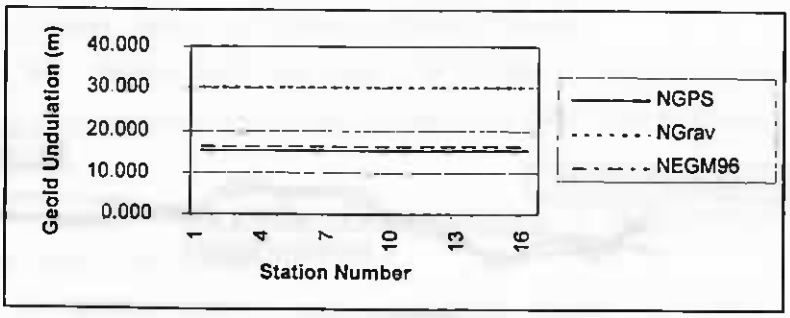

Figure 5: Plot of Geoid Undulations for the Third Test Area, Al Dabaa.

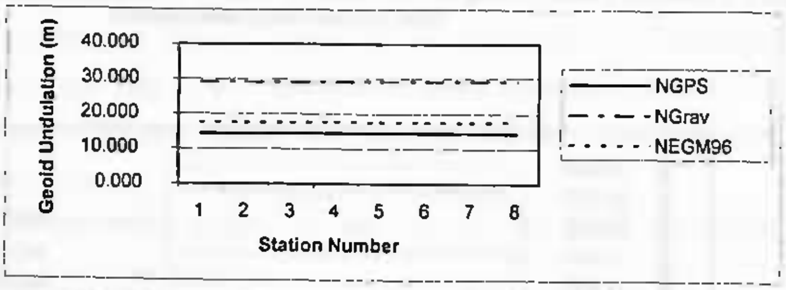

Figure 6: Plot of Geoid Undulations for the Fourth Test Area, Domiate.

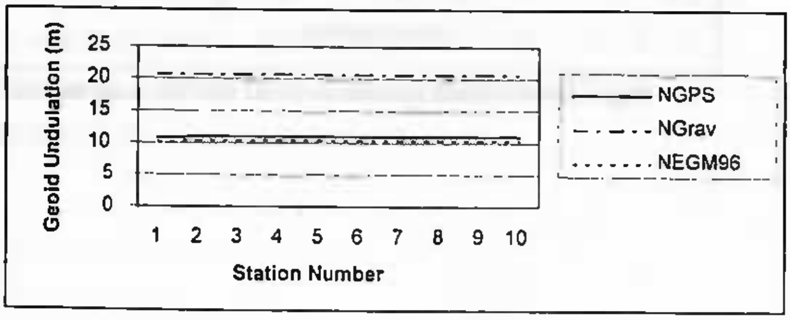

Figure 7: Plot of Geoid Undulations for the Fifth Test Area, Toshkii.

Table (1): Orthometric Heights and Geoidal Undulations Obtained From GPS/Leveling, GPS/Geoid95 and GPS/EGM96 For First Test Area, in meters.

St. No.	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
1	145.624	26.128	25.249	16.110	119.496	120.375	129.514	0.879	10.018	-0.013	-0.010
2	150.734	26.124	25.248	16.112	124.610	125.486	134.622	0.876	10.012	-0.016	-0.016
3	133.712	26.122	25.248	16.111	107.590	108.464	117.601	0.874	10.011	-0.018	-0.017
4	126.097	26.159	25.245	16.108	99.938	100.852	109.989	0.914	10.051	0.022	0.023
5	115.923	26.154	25.243	16.107	89.769	90.680	99.816	0.911	10.047	0.019	0.019
6	143.349	26.140	25.246	16.110	117.209	118.103	127.239	0.894	10.030	0.002	0.002
7	151.634	26.143	25.248	16.113	125.491	126.386	135.521	0.895	10.030	0.003	0.002
8	166.653	26.153	25.249	16.114	140.500	141.404	150.539	0.904	10.039	0.012	0.011
9	159.786	26.152	25.250	16.115	133.634	134.536	143.671	0.902	10.037	0.010	0.009
10	186.736	26.142	25.250	16.116	160.594	161.476	170.620	0.892	10.026	0.000	-0.002
11	172.893	26.159	25.259	16.125	146.734	147.634	156.768	0.900	10.034	0.008	0.006
12	174.476	26.134	25.265	16.131	148.342	149.211	158.345	0.869	10.003	-0.023	-0.025

Where:

h GPS Ellipsoidal Height,

N_{GPS} GPS/Leveling derived Geoid Height.

N_{Grav} Gravimetrically derived Geoid Height.

N_{EGM96} Geoid height derived from EGM96.

H/Lev Leveled Orthometric Height.

$H^*/_{Grav}$ O. H. from GPS and N_{Grav}.

$H^{**}/_{EGM96}$ O. H. From GPS and N_{EGM96}. .

$\Delta 1 = N_{GPS} - N_{grav}$,

$\Delta 2 = N_{GPS} - N_{EGM96}$,

$V\Delta 1 = \Delta 1$ – mean of $\Delta 1$,

$V\Delta 2 = \Delta 2$ – mean of $\Delta 2$

Table (2): Final Statistics For First Test Area Data Set, in meters.

Item	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
Minimum	115.923	26.122	25.243	16.107	89.769	90.680	99.816	0.869	10.003	-0.023	-0.025
Maximum	186.736	26.159	25.265	16.131	160.594	161.476	170.620	0.914	10.051	0.022	0.023
Mean	152.301	26.143	25.250	16.114	126.159	127.051	136.187	0.892	10.028	0.000	0.000
RMS	20.991	0.013	0.006	0.007	20.991	20.985	20.985	0.014897	0.014801	0.014897	0.014801

Table (3): Orthometric Heights and Geoidal Undulations Obtained From GPS/Leveling, GPS/Geoid95 and GPS/EGM96 For Second Test Area, in meters.

St. No.	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
1	102.772	25.308	24.396	15.990	77.464	78.376	86.782	0.912	9.318	0.013	0.018
2	86.839	25.353	24.439	16.032	61.486	62.400	70.807	0.914	9.321	0.015	0.021
3	88.229	25.356	24.470	16.071	62.873	63.759	72.158	0.886	9.285	-0.013	-0.015
4	82.008	25.251	24.377	15.972	56.757	57.631	66.036	0.874	9.279	-0.025	-0.021
5	81.898	25.262	24.361	15.963	56.636	57.537	65.935	0.901	9.299	0.002	-0.001
6	97.507	25.267	24.369	15.968	72.240	73.138	81.539	0.898	9.299	-0.001	-0.001
7	115.050	25.246	24.341	15.947	89.804	90.709	99.103	0.905	9.299	0.006	-0.001

Where:

h GPS Ellipsoidal Height, N_{GPS} GPS/Leveling derived Geoid Height, N_{Grav} Gravimetrically derived Geoid Height.

N_{EGM96} Geoid height derived from EGM96, H/Lev Leveled Orthometric Height. $H^*/_{Grav}$ O. H. from GPS and N_{Grav}.

$H^{**}/_{EGM96}$ O. H. From GPS and N_{EGM96}. . $\Delta 1 = N_{GPS} - N_{grav}$, $\Delta 2 = N_{GPS} - N_{EGM96}$,

$V\Delta 1 = \Delta 1$ – mean of $\Delta 1$, **$V\Delta 2 = \Delta 2$ – mean of $\Delta 2$**

Table (4): Final Statistics For Second Test Area Data Set, in meters.

Item	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
Minimum	81.898	25.246	24.341	15.947	56.636	57.537	65.935	0.374	9.279	-0.025	-0.021
Maximum	115.050	25.356	24.470	16.071	89.804	90.709	99.103	0.914	9.321	0.015	0.021
Mean	93.472	25.292	24.393	15.992	68.180	69.079	77.480	0.899	9.300	0.000	0.000
RMS	12.283	0.047	0.046	0.044	12.295	12.301	12.299	0.014305	0.01546	0.014305	0.01546

Table (5): Orthometric Heights and Geoidal Undulations Obtained From GPS/Leveling, GPS/Geoid95 and GPS/EGM96 For Third Test Area, in meters.

St. No.	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
1	54.002	15.105	30.362	16.372	38.897	23.640	37.630	-15.257	-1.267	-0.001	-0.004
2	54.304	15.212	30.463	16.469	39.092	23.841	37.835	-15.251	-1.257	0.005	0.006
3	58.017	15.159	30.408	16.416	42.858	27.609	41.601	-15.249	-1.257	0.007	0.006
4	66.515	15.157	30.406	16.413	51.358	36.109	50.102	-15.249	-1.256	0.007	0.007
5	67.002	15.158	30.408	16.415	51.844	36.594	50.587	-15.250	-1.257	0.006	0.006
6	67.218	15.157	30.407	16.414	52.061	36.811	50.804	-15.250	-1.257	0.006	0.006
7	66.578	15.157	30.406	16.413	51.421	36.172	50.165	-15.249	-1.256	0.007	0.007
8	54.116	15.105	30.358	16.373	39.011	23.758	37.743	-15.253	-1.268	0.003	-0.005
9	55.282	15.109	30.361	16.376	40.173	24.921	38.906	-15.252	-1.267	0.004	-0.004
10	53.545	15.104	30.358	16.371	38.441	23.187	37.174	-15.254	-1.267	0.002	-0.004
11	53.903	15.107	30.361	16.375	38.796	23.542	37.528	-15.254	-1.268	0.002	-0.005
12	52.336	15.211	30.465	16.471	37.125	21.871	35.865	-15.254	-1.260	0.002	0.003
13	50.231	15.214	30.469	16.473	35.017	19.762	33.758	-15.255	-1.259	0.001	0.004
14	53.191	15.218	30.472	16.476	37.973	22.719	36.715	-15.254	-1.258	0.002	0.005
15	51.179	15.211	30.468	16.471	35.968	20.711	34.708	-15.257	-1.260	-0.001	0.003
16	57.671	15.202	30.457	16.469	42.469	27.214	41.202	-15.255	-1.267	0.001	-0.004

Where:

h GPS Ellipsoidal Height,
N_{EGM96} Geoid height derived from EGM96,
$H^{**}/_{EGM96}$ O. H. From GPS and N_{EGM96}. .
$V\Delta 1 = \Delta 1$ – mean of $\Delta 1$,

N_{GPS} GPS/Leveling derived Geoid Height,
H/Lev Leveled Orthometric Height,
$\Delta 1 = N_{GPS} - N_{grav}$,
$V\Delta 2 = \Delta 2$ – mean of $\Delta 2$

N_{Grav} Gravimetrically derived Geoid Height
$H^*/_{Grav}$ O. H. from GPS and N_{Grav}
$\Delta 2 = N_{GPS} - N_{EGM96}$,

Table (6): Final Statistics For Third Test Area Data Set, in meters.

Item	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
Minimum	50.231	15.104	30.358	16.371	35.017	19.762	33.758	-15.257	-1.268	-0.001	-0.005
Maximum	67.218	15.218	30.472	16.476	52.061	36.811	50.804	-15.249	-1.256	0.007	0.007
Mean	57.193	15.162	30.414	16.423	42.032	26.779	40.770	-15.253	-1.261	0.003	0.002
RMS	8.539	0.045	0.045	0.042	6.076	6.078	6.079	0.003	0.004963	0.00275	0.004963

Table (7): Orthometric Heights and Geoidal Undulations Obtained From GPS/Leveling, GPS/Geoid95 and GPS/EGM96 For Fourth Test Area, in meters.

St. No.	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^{*}/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
1	18.337	14.416	29.144	17.601	3.921	-10.807	0.736	-14.728	-3.185	-0.016	0.003
2	18.724	14.390	29.129	17.583	4.338	-10.405	1.141	-14.739	-3.193	-0.027	-0.005
3	17.949	14.473	29.172	17.652	3.476	-11.223	0.297	-14.699	-3.179	0.013	0.009
4	17.859	14.483	29.180	17.661	3.376	-11.321	0.198	-14.697	-3.178	0.015	0.010
5	18.151	14.468	29.163	17.639	3.684	-11.012	0.512	-14.695	-3.171	0.017	0.017
6	16.278	14.476	29.171	17.642	1.802	-12.893	-1.364	-14.695	-3.166	0.017	0.022
7	17.955	14.452	29.169	17.624	3.503	-11.214	0.331	-14.717	-3.172	-0.005	0.016
8	18.023	14.402	29.131	17.661	3.621	-11.108	0.362	-14.729	-3.259	-0.017	-0.071

Where:

h	GPS Ellipsoidal Height,	N_{GPS}	GPS Leveling derived Geoid Height,	N_{Grav}	Gravimetrically derived Geoid Height,
N_{EGM96}	Geoid height derived from EGM96,	H/Lev	Leveled Orthometric Height,	$H^{*}/_{Grav}$	O. H. from GPS and N_{Grav}.
$H^{**}/_{EGM96}$	O. H. From GPS and N_{EGM96}		$\Delta 1 = N_{GPS} - N_{Grav}$,		$\Delta 2 = N_{GPS} - N_{EGM96}$,

$V\Delta 1 = \Delta 1$ – mean of Δ1, $V\Delta 2 = \Delta 2$ – mean of Δ2

Table (8): Final Statistics For Fourth Test Area Data Set, in meters.

Item	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^{*}/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
Minimum	16.278	14.390	29.129	17.583	1.802	-12.893	-1.364	-14.739	-3.259	-0.027	-0.071
Maximum	18.724	14.483	29.180	17.661	4.338	-10.405	1.141	-14.695	-3.166	0.017	0.022
Mean	17.910	14.445	29.157	17.633	3.465	-11.248	0.277	-14.712	-3.188	0.000	0.000
RMS	0.716	0.037	0.020	0.028	0.738	0.726	0.729	0.018007	0.029964	0.018007	0.029964

Table (9): Orthometric Heights and Geoidal Undulations Obtained From GPS/Leveling, GPS/Geoid95 and GPS/EGM96 For Fifth Test Area, in meters.

St. No.	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
1	246.186	10.863	20.705	10.361	235.323	225.481	235.825	-9.842	0.502	-0.0106	-0.0107
2	217.754	10.896	20.727	10.392	206.858	197.027	207.362	-9.831	0.504	0.0004	-0.0087
3	213.811	10.916	20.741	10.401	202.895	193.07	203.41	-9.825	0.515	0.0064	0.0023
4	215.236	10.937	20.767	10.421	204.299	194.469	204.815	-9.83	0.516	0.0014	0.0033
5	224.944	10.952	20.789	10.443	213.992	204.155	214.501	-9.837	0.509	-0.0056	-0.0037
6	223.335	10.972	20.802	10.459	212.363	202.533	212.876	-9.83	0.513	0.0014	0.0003
7	216.766	10.992	20.826	10.482	205.774	195.94	206.284	-9.834	0.51	-0.0026	-0.0027
8	224.521	11.006	20.837	10.491	213.515	203.684	214.03	-9.831	0.515	0.0004	0.0023
9	216.999	11.026	20.854	10.509	205.973	196.145	206.49	-9.828	0.517	0.0034	0.0043
10	205.984	11.047	20.873	10.521	194.937	185.111	195.463	-9.826	0.526	0.0054	0.0133

Where:

h GPS Ellipsoidal Height,

N_{EGM96} Geoid height derived from EGM96,

$H^{**}/_{EGM96}$ O. H. From GPS and N_{EGM96} ,

VΔ1 = Δ1 – mean of Δ1,

N_{GPS} GPS/Leveling derived Geoid Height,

H/Lev Leveled Orthometric Height,

Δ1 = N_{GPS} - N_{Grav} ,

VΔ2 = Δ2 – mean of Δ2

N_{Grav} Gravimetrically derived Geoid Height,

$H^*/_{Grav}$ O. H. from GPS and N_{Grav},

Δ2 = N_{GPS} - N_{EGM96} ,

Table (10): Final Statistics For Fifth Test Area Data Set, in meters.

Item	h	N_{GPS}	N_{Grav}	N_{EGM96}	H/Lev	$H^*/_{Grav}$	$H^{**}/_{EGM96}$	Δ1	Δ2	V/Δ1	V/Δ2
Minimum	205.984	10.863	20.705	10.361	194.937	185.111	195.463	-9.842	0.502	-0.0106	-0.0107
Maximum	246.186	11.047	20.873	10.521	235.323	225.481	235.825	-9.825	0.526	0.0064	0.0133
Mean	220.5536	10.9607	20.7921	10.448	209.5929	199.7615	210.1056	-9.8314	0.5127	-5.32E-15	-5.66E-15
RMS	10.64896	0.058914	0.056485	0.053587	10.68382	10.679404	10.678716	0.005125	0.006897	0.0051251	0.0068968